

HBTU Logo	Harcourt Butler Technical University, Kanpur		MID SEM I
Branch	CS/ET/IT/CE/EE/ME	Program	B.Tech.
Course Name	Engineering Mathematics-I	Semester	II
Course Code	NMA 102	Year/Session	2025-26
Time: 1 hr		Max. Marks	15
Knowledge Level (KL)	K1: Remembering K2: Understanding	K3: Applying K4: Analysing	K5: Evaluating K6: Creating

Note: Attempt any five questions. All questions carry equal marks.

Q. No	Questions	Marks	COs	KL
1	If $y = \sin[\log(x^2 + 2x + 1)]$, prove that $(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2 + 4)y_n = 0$.	3	CO1	K3
2	Expand $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$ in the powers of $(x - 1)$ and find $f(0.99)$.	3	CO1	K3
3	Discuss the convergence of the improper integral $\int_{-\infty}^{\infty} x e^{-x^2} dx$.	3	CO1	K4
4	If $a^2 x^2 + b^2 y^2 = c^2 z^2$, evaluate $\frac{1}{a^2} \frac{\partial^2 z}{\partial x^2} + \frac{1}{b^2} \frac{\partial^2 z}{\partial y^2}$.	3	CO2	K5
5	If $u = \frac{x^3 y^3}{x^3 + y^3}$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3u$.	3	CO2	K4
6	Find the area enclosed between the curve $x^2 = 4y$ and the line $x = 4y - 2$.	3	CO2	K5

Engineering Mathematics I: Mid-sem 1, 2025–26 (Semester II)

Model answers prepared for study; not an official university answer key.

1. Derivative recurrence

Write $t = x + 1$ and $y = \sin(\log t^2)$ for $t \neq 0$. Differentiation gives $t^2 y'' + t y' + 4y = 0$. Differentiate n times using Leibnitz's rule:

$$t^2 y_{n+2} + 2nt y_{n+1} + n(n-1)y_n + t y_{n+1} + n y_n + 4y_n = 0.$$

Thus $\boxed{(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0}$.

2. Expansion and numerical value

Taylor's theorem about $x = 1$ gives an exact finite expansion because the polynomial has degree five:

$$f(x) = \sum_{k=0}^5 \frac{f^{(k)}(1)}{k!} (x-1)^k.$$

The derivative values are $f(1) = 0$, $f'(1) = 3$, $f''(1) = 12$, $f^{(3)}(1) = 42$, $f^{(4)}(1) = 96$ and $f^{(5)}(1) = 120$. Put $h = x - 1$ and substitute:

$$\boxed{f(x) = 3h + 6h^2 + 7h^3 + 4h^4 + h^5}.$$

For $x = 0.99$, $h = -0.01$, and

$$f(0.99) = -0.03 + 0.0006 - 0.000007 + 0.00000004 - 0.0000000001 = \boxed{-0.0294069601}.$$

3. Improper integral

The usual improper integral is defined by the sum of the two separate integrals at any finite splitting point. Since $\int x e^{-x^2} dx = -\frac{1}{2} e^{-x^2}$,

$$\int_{-\infty}^0 x e^{-x^2} dx = -\frac{1}{2}, \quad \int_0^{\infty} x e^{-x^2} dx = \frac{1}{2}.$$

Moreover $\int_{-\infty}^{\infty} |x| e^{-x^2} dx = 1$. It therefore converges absolutely and its value is $\boxed{0}$, not merely a Cauchy principal value.

4. Second partial derivatives

Assume nonzero constants a, b, c and a differentiable branch with $z \neq 0$. Implicit differentiation gives

$$z_x = \frac{a^2 x}{c^2 z}, \quad z_y = \frac{b^2 y}{c^2 z}, \quad z_{xx} = \frac{a^2}{c^2 z} - \frac{a^4 x^2}{c^4 z^3}, \quad z_{yy} = \frac{b^2}{c^2 z} - \frac{b^4 y^2}{c^4 z^3}.$$

Therefore

$$\frac{z_{xx}}{a^2} + \frac{z_{yy}}{b^2} = \frac{2}{c^2 z} - \frac{a^2 x^2 + b^2 y^2}{c^4 z^3} = \boxed{\frac{1}{c^2 z}}.$$

5. Euler relation

Let $D = x^3 + y^3 \neq 0$. Quotient differentiation gives $u_x = 3x^2 y^6 / D^2$, $u_y = 3y^2 x^6 / D^2$. Hence

$$x u_x + y u_y = \frac{3x^3 y^3 (x^3 + y^3)}{D^2} = \boxed{3u}.$$

Equivalently u is homogeneous of degree three.

6. Enclosed area

The line is $y = (x + 2)/4$ and the parabola is $y = x^2/4$. Their intersections satisfy $x^2 = x + 2$, so $x = -1, 2$. The difference between the line and parabola is $(x + 2 - x^2)/4 = -(x - 2)(x + 1)/4$, which is positive throughout $(-1, 2)$. Thus the line is the upper boundary over the whole interval. A vertical strip has area $(y_{\text{upper}} - y_{\text{lower}}) dx$, so

$$A = \frac{1}{4} \int_{-1}^2 (x + 2 - x^2) dx = \frac{1}{4} \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^2 = \boxed{\frac{9}{8} \text{ square units}}.$$